MTH 264 Introduction to Matrix Algebra. Summer 2023.. Spring 2023

HW3B. Written Homework 3B. **Due Week 3 Friday 11:59PM**

Name:

Instructions: Upload a pdf of your submission to **Gradescope**. This worksheet is worth 20 points: up to 8 points will be awarded for accuracy of certain parts (to be determined after the due date) and up to 12 points will be awarded for completion of parts not graded by accuracy.

For each of the following linear transformations T, find the matrix $\mathbf{A}$ such that $T(\mathbf{x}) = \mathbf{A}\mathbf{x}$.

Remark: You don't need to show the steps for matrix operation calculations. e.g. When finding the inverse $\mathbf{A}^{-1}$, there's no need to show how you got the inverse. Just label the corresponding matrix appropriately.

(1) The transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ such that $T\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\2\\3\end{pmatrix}$ and $T\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}3\\4\\5\end{pmatrix}$

(2) The transformation $S : \mathbb{R}^3 \to \mathbb{R}^2$ such that $S(\mathbf{e_1}) = \begin{pmatrix}1\\2\end{pmatrix}$, $S(\mathbf{e_2}) = \begin{pmatrix}3\\4\end{pmatrix}$, and $S(\mathbf{e_3}) = \begin{pmatrix}5\\6\end{pmatrix}$

(3) The **(left) dot product** $D_L : \mathbb{R}^3 \to \mathbb{R}$ given by $\mathbf{x} \mapsto \mathbf{x} \cdot \mathbf{a}$ with $\mathbf{a} \in \mathbb{R}^3$ some fixed vector, i.e.

$$D_L\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} = \begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} \cdot \mathbf{a} = a_1x_1 + a_2x_2 + a_3x_3 \qquad \text{with} \qquad \mathbf{a} = \begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix}$$

Remark: To find the matrix $\mathbf{A}$, we denote the result of the dot product (i.e. a scalar) using a (1×1)-matrix.

(4) The **(right) cross product** $C_R : \mathbb{R}^3 \to \mathbb{R}^3$ given by $\mathbf{x} \mapsto \mathbf{b} \times \mathbf{x}$ with $\mathbf{b} \in \mathbb{R}^3$ some fixed vector, i.e.

$$C_R\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} = \mathbf{b} \times \begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} = \begin{pmatrix}b_2x_3 - b_3x_2\\b_3x_1 - b_1x_3\\b_1x_2 - b_2x_1\end{pmatrix} \qquad \text{with} \qquad \mathbf{b} = \begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix}$$

(5) The **rotation** $R_\theta : \mathbb{R}^2 \to \mathbb{R}^2$ rotating the plane by $\frac{\pi}{3}$ radians, counterclockwise.

(6) The transformation $Q : \mathbb{R}^2 \to \mathbb{R}^2$ such that $Q\begin{pmatrix}1\\2\end{pmatrix} = \begin{pmatrix}0\\2\end{pmatrix}$ and $Q\begin{pmatrix}3\\4\end{pmatrix} = \begin{pmatrix}2\\0\end{pmatrix}$

(7) The **reflection** $R_1 : \mathbb{R}^2 \to \mathbb{R}^2$ on $\mathbb{R}^2$ across the line spanned by the vector $\mathbf{y} = \begin{pmatrix}3\\-4\end{pmatrix}$.

Hint: The set $V = \left\{ \begin{pmatrix}3\\-4\end{pmatrix}, \begin{pmatrix}-4\\-3\end{pmatrix} \right\}$ is a convenient basis for $\mathbb{R}^2$.

(8) The **orthogonal projection** $O_1 : \mathbb{R}^2 \to \mathbb{R}^2$ on $\mathbb{R}^2$ on the line $y = \dfrac{1}{2}x$.

Hint: The set $V = \left\{ \begin{pmatrix}2\\1\end{pmatrix}, \begin{pmatrix}-1\\2\end{pmatrix} \right\}$ is a basis for $\mathbb{R}^2$.

(9) The **reflection** R_2 on $\mathbb{R}^3$ across the plane $P_2 = \{(x, y, z) \in \mathbb{R}^3 : 2x + 3y + z = 0\}$.

Hint: The set $V = \left\{ \begin{pmatrix}2\\3\\1\end{pmatrix}, \begin{pmatrix}3\\-2\\0\end{pmatrix}, \begin{pmatrix}0\\-1\\3\end{pmatrix} \right\}$ is a basis for $\mathbb{R}^3$.

(10) The **reflection** R_3 on $\mathbb{R}^3$ across the plane $P_3 = \left\{(x, y, z) \in \mathbb{R}^3 : -x + 2y + z = 0\right\}$.

Hint: Let $\mathbf{N}$ be the normal vector for P_3 and let $\mathbf{d_1}, \mathbf{d_2}$ some linearly independent set of direction vectors for P_3. Then, consider the basis $\{\mathbf{N}, \mathbf{d_1}, \mathbf{d_2}\}$ for $\mathbb{R}^3$.

(11) The **orthogonal projection** O_2 on $\mathbb{R}^3$ across the plane P_4 given by $P(t, s) = t \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + s \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$.

Hint: Let $\mathbf{N}$ be the normal vector for P_4 and let $\mathbf{d_1}, \mathbf{d_2}$ some linearly independent set of direction vectors for P_4. Then, consider the basis $\{\mathbf{N}, \mathbf{d_1}, \mathbf{d_2}\}$ for $\mathbb{R}^3$.

(12) The transformation $T_1 : \mathbb{R}^2 \to \mathbb{R}^2$ that accepts a vector $\mathbf{x} \in \mathbb{R}^2$, rotates it by $\frac{\pi}{3}$ radians, counterclockwise, and then reflects it across the line $y = -\frac{4}{3}x$.

Hint: Consider the transformations from Problems (5) and (7).

(13) The transformation $T_2 : \mathbb{R}^2 \to \mathbb{R}^2$ that accepts a vector $\mathbf{x} \in \mathbb{R}^2$, reflects it across the line $y = -\frac{4}{3}x$, and then rotates it by $\frac{\pi}{3}$ radians, counterclockwise.

Hint: Consider the transformations from Problems (5) and (7).